SOLUTIONS TO SELECTED QUESTIONS IN HOMEWORK 6

MATH 241

18.4.11

Proof. Let $f(z) := e^{z^2}$, and $f'(z) = 2ze^{z^2}$, so $f''(z) = 2e^{z^2} + 4z^2e^{z^2}$, and $f''(i) = 2e^{i^2} + 4(i)^2e^{i^2} = -2e^{-1}$. By Cauchy's integral formula,

$$\oint_C \frac{e^{z^2}}{(z-i)^3} = \frac{2\pi i}{2!}(-2e^{-1}) = -2e^{-1}\pi i$$

□

18.4.23

Proof. C splits into two simple closed curves, we call the left one around 0 as C_1, and the right one around 2 as C_2. Notice that C_1 is oriented clockwisely, so you should have an opposite sign in front of the usual formula.

$$\oint_{C_1} \frac{3z+1}{z(z-2)^2}dz = -2\pi i\frac{3\cdot 0+1}{(0-2)^2} = -\frac{\pi i}{2}$$

$$\oint_{C_2} \frac{3z+1}{z(z-2)^2}dz = 2\pi i(\frac{3z+1}{z})'(2) = 2\pi i(-\frac{1}{z^2})(2) = -\frac{\pi i}{2}$$

So add up to

$$\oint_C \frac{3z+1}{z(z-2)^2}dz = -\pi i$$

□

Fall 09, #2

Proof. Decompose the contour into three, one includes -1 and 1, counterclockwise, one includes -1, clockwise, and one includes 1, counterclockwise. So by Cauchy-Goursat Theorem for multi-connected domain you end up with 2 times of the contour integral around 1, counterclockwise. So eventually you get

$$2\oint_{C_1} \frac{\frac{1}{z+1}}{z-1}dz = 2\cdot 2\pi i\frac{1}{1+1} = 2\pi i$$

□